Nonlinear Electrical Predistortion and Equalization for the Coherent Optical Communication System

Jie Pan and Chi-Hao Cheng

***Abstract*—One of major issues of the advanced optical communication system is the signal distortion caused by fiber nonlinearity. Both predistorters and equalizers have been used for nonlinearity compensation and electrical compensation has become a popular choice among optical communication engineers in recent years. However, to the best of the authors' knowledge, no comparative study about electrical equalizers and predistorters has been conducted for optical communication systems. Using a coherent optical orthogonal frequency division multiplexing (CO-OFDM) system as a test system, we investigated the performance of electrical pth-order inverse Volterra predistorters and equalizers. Our research results show that both the equalizer and the predistorter can compensate for the nonlinearity of the CO-OFDM system. The major difference between the predistorter and the equalizer is that the predistorter regulates the input power; therefore, the performance of the CO-OFDM system with a predistorter is independent over a wide range of the laser launch power. The results presented in this paper can lead to a better understanding of electrical equalization and predistortion techniques for optical communication systems.**

***Index Terms*—Equalizer, nonlinear distortion, orthogonal frequency-division multiplexing (OFDM), optical fiber communication, predistorter, pth-order inverse, Volterra series.**

I. Introduction

NONLINEAR signal distortion caused by fiber nonlinearities such as *self-phase modulation* (SPM) and *four-wave mixing* (FWM) is one of the major performance-limiting factors of advanced optical communication systems [1]. Both predistortion and equalization have been applied to compensate for nonlinear signal distortions [2]–[5]. The difference between an equalizer and a predistorter is that the equalizer compensates for the signal distortion at the receiver and the predistorter precompensates for the signal at the transmitter. The equalizer has been demonstrated to be capable of compensating for the joint effects of intersymbol interference, nonlinearities, and noise at the receiver [2]. A potential issue associated with the equalizer is that it may amplify noise whereas compensating for signal distortion [2]. The predistorter can circumvent the noise enhancement problem and still compensate for the nonlinear signal distortion before the addition of noise; however, the predistorter would vary the actual input signal of the nonlinear system, making the channel model, on which the predistorter design is based, inaccurate [3], [4]. In this paper, we investigate the applications of electrical equalizers and predistorters for coherent optical communication system nonlinearity compensation. To the best of authors' knowledge, this is the first attempt to compare a predistorter and an equalizer in such an application.

The equalizer and predistorter investigated in this study are designed based on a pth-order inverse Volterra system [6]. The Volterra paradigm [7] has been used to model the optical communication system nonlinearity [8]–[12], to mitigate nonlinearity effects in optical communication systems [13], and to design electrical equalizers for optical systems [11], [14]–[18]. According to the pth-order inverse theory, the nonlinearity in the pth-order Volterra system can be compensated up to the pth-order by another pth-order Volterra system referred to as the pth-order inverse system. The pth-order inverse theory can be applied to design the equalizer and predistorter. The pth-order inverse is chosen as the compensation scheme in this study because it can be easily used to design an equalizer and predistorter.

We applied the pth-order inverse electrical equalizer and predistorter to compensate for the nonlinear signal distortion of a 16 quadratic-amplitude modulation (QAM), 100 Gb/s *coherent optical orthogonal frequency division multiplexing* (CO-OFDM) system and investigate their performance difference. The CO-OFDM is considered a potential candidate for the next generation optical communication system [19]. One of major issues of the CO-OFDM system is its vulnerability to fiber nonlinear effects such as SPM, *cross-phase modulation*, and the FWM among subcarriers [14], [15]. The major focus of this paper is to design the electrical equalizer/predistorter to compensate intrachannel nonlinearity such as SPM and FWM among subcarriers of a CO-OFDM system. Although the developed electrical compensators are designed for single channel OFDM system, our previous works demonstrate that such a compensator can still improve a wavelength division multiplexing OFDM system's performance [18]. Compared with the back-propagation scheme [20]–[22], a popular compensation method which can compensate interchannel and intrachannel nonlinearities, the compensation scheme considered in this paper has simpler structure at the cost of limited compensation capability. Since each channel needs one pth-order inverse equalizer/predistorter and the equalizer/predistorter of each channel works independently, the pth-order inverse equalizer/predistorter is highly scalable. On the other hand, since the back-propagation equalization needs to take signals from

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every channel into account, it might not be feasible for high channel-count systems.

The rest of this paper is organized as follows. The Volterra model and the pth-order inverse theory are introduced in Section II, the CO-OFDM system simulation diagram used in this study is described in Section III, the simulation results and discussions are presented in Section IV, and Section V concludes this paper.

II. Volterra Model and p-th Order Inverse

The Volterra series can be considered as a Taylor series with memory and the input–output relation of a discrete-time Volterra model is given as follows:

$$\begin{aligned} y(n) &= \sum_{i=0}^{\infty} h_1(i)x(n-i) \\ &\quad + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} h_2(i,j)x(n-i)x(n-j) \\ &\quad + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} h_3(i,j,k) \\ &\quad \times x(n-i)x(n-j)x(n-k) + \cdots \end{aligned} \tag{1}$$

where $x[\cdot]$ is the input signal, $y[\cdot]$ is the output signal, and $h_i[\cdot]$ is the ith-order kernels of the Volterra model.

Because of the bandpass nature of the communication channel [23], the input and output signals of a communication system are often represented by its signals' complex envelopes. When the Volterra model is used to model the complex envelope input–output relation of a bandpass system, the even-order Volterra kernels are ignored because they do not generate in-band signals. A discrete causal third-order bandpass Volterra model with finite memory length can be represented using the following equation [24]:

$$\begin{aligned} y(n) &= \sum_{i=0}^{N} h_1(i)x(n-i) \\ &\quad + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{k=0}^{N} h_3(i,j,k)x(n-i)x(n-j)x^*(n-k) \end{aligned} \tag{2}$$

where N is the memory length, $(\,)^*$ denotes the complex conjugate, $x(n)$ and $y(n)$ are the input and output signal complex envelopes, and $h_1(i)$ and $h_3(i,j,k)$ are the linear and cubic Volterra kernels, respectively.

The input–output relation of an Mth-order Volterra system H can be represented as

$$y(n) = H[x(n)] = \sum_{i=1}^{M} H_i[x(n)] \tag{3}$$

$$H_i[x(n)] = \sum_{k_1=0}^{N} \cdots \sum_{k_i=0}^{N} h_i(k_1,\ldots,k_i)x(n-k_1)\cdots x(n-k_i) \tag{4}$$

where H is the system operator, H_i is the ith-order Volterra operator, and $h_i[\cdot]$ is the ith-order kernel of the Volterra system.

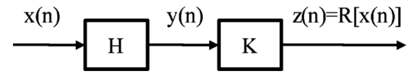

Fig. 1. Nonlinear system $\boldsymbol{H}$ connected with its inverse system $\boldsymbol{K}$ (pth-order inverse system).

As shown in Fig. 1, K is the pth-order inverse system of the nonlinear system H, and R represents the cascading system. The pth-order inverse of H is defined as a pth-order nonlinear system that, when connected in tandem with the nonlinear system H, will result in a cascading system R whose second- to pth-order Volterra operators are zero [6].

The input–output relation of pth-order inverse system K can be written as in (5), where K_i is the ith-order Volterra operator:

$$K[y(n)] = \sum_{i=1}^{p} K_i[y(n)]. \tag{5}$$

It has been shown that a third-order bandpass Volterra system is capable of modeling and compensating a coherent optical communication system [8], [18]. Our goal is to obtain the third-order inverse system of the third-order bandpass Volterra system. Due to its bandpass nature, the second-order operator of the inverse system can be ignored. The first- and third-order operator of the third-order inverse filter can be derived as follows [6]:

$$K_1 = H_1^{-1} \tag{6}$$

$$K_3 = -K_1 H_3 K_1. \tag{7}$$

The pth-order inverse system followed by the nonlinear system is referred to as the pth-order predistorter, and the pth-order inverse system following the nonlinear system is referred to as the pth-order equalizer. The pth-order inverse equalizer and the pth-order inverse predistorter are identical [6]. In our study, we used a third-order bandpass Volterra model to model a CO-OFDM system and then, based on the derived model, designed its third-order inverse equalizer and predistorter using pth-order inverse theory.

III. System Description and Modeling

We applied the CO-OFDM system simulation setup used in our previous electrical equalizer research work in this equalization/predistortion study [18]. The CO-OFDM system, whose configuration is illustrated in Fig. 2, is simulated by a commercial fiber-optic system simulation tool, OptiSystem 8.0.

The data transmission rate of this CO-OFDM system is 100 Gb/s, and its modulation scheme is 16-QAM. The frequency of the carrier is set at 193.1 THz. The optical channel consists of ten spans of 80 km *standard single mode fiber* (SSMF) whose dispersion is fully compensated by the *dispersion compensation fiber* (DCF) in each span. The attenuation of SSMF and DCF is compensated by an optical amplifier in each loop. Transmitted bits are saved and compared with received bits at the end of the receiver to determine *bit error rate* (BER).

IV. Results and Discussions

Our previous research results show that a third-order Volterra equalizer with memory length of 2 can be used to compensate

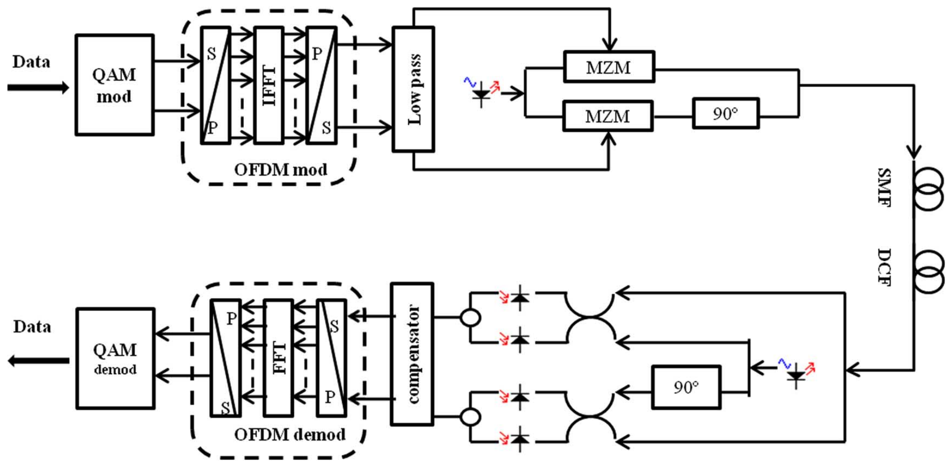

Fig. 2. Block diagram of CO-OFDM [18].

the CO-OFDM system described in the previous section [18]. It is noteworthy that, in the CO-OFDM system under consideration, the fiber dispersion is fully compensated by the DCF whose nonlinearity is more significant than SSMF. Therefore, a third-order Volterra system should be sufficient for a dispersion-unmanaged CO-OFDM system with an electronic dispersion compensator at the receiver [25]. In this scenario, two electronic processors (one for dispersion compensation and one for nonlinear compensation) might be needed. An alternative is that these two devices be combined as one device.

To design the third-order inverse equalizer and predistorter, we need to obtain the third-order Volterra channel model first. This can be accomplished by transmitting a predetermined training sequence and estimating the dependence of the channel output signals on the training sequence [3]. We used a third-order bandpass Volterra model with a memory length of two to model the CO-OFDM system illustrated in Fig. 2 at different laser launch powers using *recursive least square* methods [26]. A third-order inverse equalizer is then determined based on the system model. For comparison purpose, we also used a *finite-impulse response* (FIR) linear filter to model the same system and then designed the equalizer based on the linear inverse of the FIR filter.

Since the pth-order inverse predistorter changes the nonlinear system input, the nonlinear model on which the pth-order inverse predistorter is based might not be accurate after the predistorter is introduced. The reason is the nonlinear system characteristics might depend on the input signal [3]. To alleviate this problem, we need to conduct the system modeling and the corresponding pth-order predistorter design over several iterations. The detailed design procedure of a pth-order inverse predistorter is given as follows.

Step 1: Determine the Volterra channel model H in the training mode.

Step 2: Develop the pth-order inverse system K based on the Volterra channel model H obtained in Step 1. Notice that the pth-order inverse system can be either an equalizer or a predistorter.

Step 3: Use the predistorter output on the original training sequence as the new training sequence and perform the system modeling again. Update the channel model H.

Step 4: Develop the pth-order inverse predistorter K based on the new channel model H.

Step 5: Go to Step 3. End if the performance of the new predistorter derived in Step 4 levels off.

In principle, the Volterra model of each channel can be determined periodically, and we can determine the equalizer and the predistorter accordingly. We used an 8192 bit training sequence to determine the channel model. In a 100 Gb/s system, it takes 0.08192 μs to update the link status provided that the compensation circuit can be implemented as a real-time device. The predistorter might take a longer time to update since the predistorter changes the actual input signal and it might be necessary to go through a couple of iterations ($\cong 0.164\ \mu$s) to update the predistorter.

The derived third-order equalizer and predistorter are then used to compensate different sequences whose total length is 2^{16} bits. The comparison of BER versus laser launch power of the OFDM systems, with and without compensation, is shown in Fig. 3. For the system without predistorters, low/high laser launch power means low/high input signal power entering the fiber; however, it is not true when a predistorter is used as we will describe later. For the system without compensation or with equalizer compensation (the predistorter case will be discussed later), the BER has a parabolic tendency and the BER will reach a minimum point at a certain laser launch power. The explanation is that under low input power level, the fiber nonlinearity effect is weak and the low *optical signal-to-noise ratio* (OSNR) limits the equalizer performance. When the input signal power increases, the system BER decreases at first due to the improved OSNR and then increases when the input signal power is larger than the "optimal" power because of the increased system nonlinear distortion under high input power. As shown in Fig. 3, OFDM systems with (or without) different equalizers have different "optimal" power and BER values. The OFDM system with an equalizer can handle higher power and reach lower BER. The lowest BER occurs at around -3 dBm laser launch power for the system without compensation. With linear inverse equalization, the system's lowest BER occurs around -1 dBm laser launch power. With third-order inverse equalization, the system's lowest BER occurs around 0 dBm laser launch power. The system with the nonlinear equalizer can take higher laser

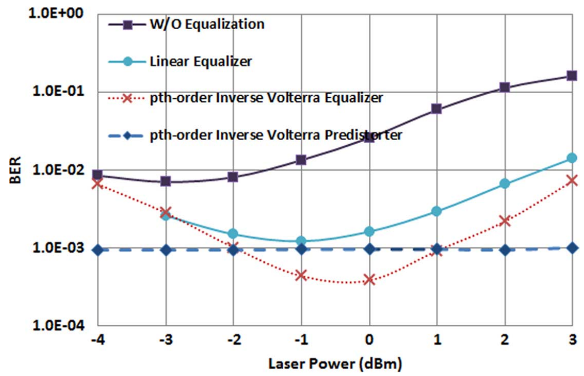

Fig. 3. BER of the CO-OFDM systems without an equalizer, with linear equalizer, with third-order Volterra inverse equalizer, and with third-order Volterra inverse predistorter at different laser launch power.

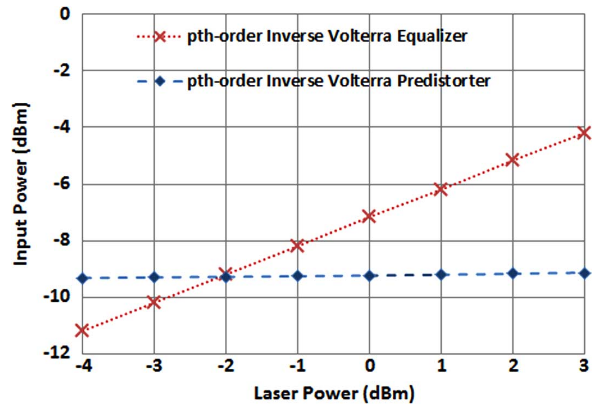

Fig. 4. Input signal power entering the fiber of CO-OFDM systems with third-order inverse equalizer and with third-order inverse predistorter at different laser launch power.

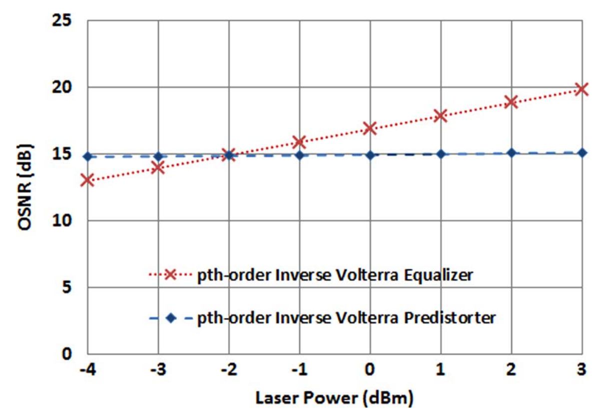

Fig. 5. OSNR of the CO-OFDM system with third-order inverse equalizer and with third-order inverse predistorter at different laser launch power.

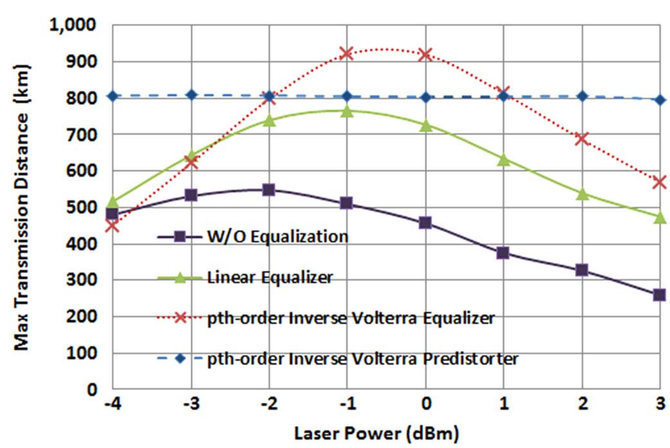

Fig. 6. Maximum fiber transmission lengths versus laser launch power of the OFDM systems without compensation, with linear inverse equalizer, with third-order inverse equalizer, and with third-order inverse predistorter.

launch power and achieve lower BER compared with the one with the linear equalizer or without compensation.

One might notice that, under high laser launch power, the linear and Volterra equalizers have comparable performances although the channel has become highly nonlinear. Our explanation is given as follows. Under high laser launch power, the SPM and intrachannel FWM severely deteriorate the signals. Thus, neither linear nor nonlinear model can predict the channel behavior accurately. The Volterra equalizer includes the polynomial operation of noise at receiver. When the nonlinear channel model is not accurate, an additional "polynomial noise" term will be generated. However, for a linear equalizer, this effect is weaker.

Fig. 3 also shows that the best performance is delivered by the Volterra equalizer rather than Volterra predistorter and, different from the system with an equalizer, the system with the third-order inverse predistorter maintains an almost constant BER value at different laser launch power. To explain these phenomena, we compare the actual input signal power entering the fiber under different laser launch powers. For the system with the third-order inverse equalizer, since the equalization is performed at the receiver, the input signal power will not be affected by the equalizers and would increase with the increase of the laser launch power. However, the third-order inverse predistorter changes signals at the transmitter so it acts like an input power regulator. As a result, for the system with a third-order inverse predistorter, its input signal power changes slightly, maintaining a nearly constant level when the laser launch power increases from -4 to 3 dBm as shown in Fig. 4.

The OSNR at the end of transmission is calculated and shown in Fig. 5. The OSNR of the system with the third-order inverse equalizer increases as the laser launch power increases, while the OSNR of the system with the third-order inverse predistorter increases only slightly. The reason is that the input signal power after predistorter will be maintained at around the same level for different laser launch powers. Thus, the OSNR at the end of transmission does not change significantly. This result is consistent with Fig. 4.

Based on the simulation results presented in Figs. 4 and 5, we conclude that, as shown in Fig. 3, the best performance is achieved by the Volterra inverse equalizer rather than the predistorter because the predistorter fixes the actual input power at a value which does not deliver the best performance. Under the same input signal power, the equalizer and predistorter deliver similar performance. The optimal method for designing the predistorter such that it can fix the input signal power at any level demands further investigation.

Fig. 6 shows the CO-OFDM systems' maximum possible transmission length at different laser launch powers to guarantee a 10^{-3} BER in simulations. The maximum fiber transmission length of the system with third-order Volterra inverse predistorter remains around 800 km under different laser launch power and the maximum fiber transmission length of the OFDM system with an equalizer or without compensation is laser launch power dependent. This result is consistent with the results shown in Figs. 3–5.

V. Conclusion

This paper presents the results of an investigation on electrical equalizers and predistorters for CO-OFDM system compensation. Based on the third-order Volterra model of a CO-OFDM system, we successfully derived a third-order inverse predistorter and equalizer to compensate for CO-OFDM system nonlinear distortion. Our simulation results show that the predistorter acts as an input signal power regulator and the power level of the signal entering the fiber is maintained at around the same power at different laser launch power. Therefore, the BER and the maximum transmission fiber length of the CO-OFDM system with a third-order inverse predistorter remain roughly constant under different laser launch power. As a result, although the pth-order inverse predistorter and equalizer have similar performance under the same input power, they have different performance under different laser launch power. The results presented in this paper can serve as a guideline for optical engineers to select between predistorters and equalizers in their applications.

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